

# The influence of Earth rotation in neutrino speed measurements between CERN and the OPERA detector

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## Abstract

The OPERA neutrino experiment at the underground Gran Sasso Laboratory recently reported, in arXiv:1109.4897v1, high-accuracy velocity measurements of neutrinos from the CERN CNGS beam over the 730 km distance between the two laboratories. This raised significant interest, as the observed neutrinos appeared to arrive at the OPERA detector about 60 ns (or equivalently 18 m) earlier than would have been expected if they had traveled at the speed of light, with high statistical significance. As the authors did not indicate whether and how they took into account the Coriolis or Sagnac effect that Earth's rotation has on the (southeastwards traveling) neutrinos, this brief note quantifies this effect. It would explain a 2.2 ns later arrival time.

The speed of light in vacuum is  $c = 299\,792\,458$  m/s in inertial frames of reference, that is, coordinate systems that do not rotate or accelerate. However, in the ground-based neutrino-speed measurements reported in [1], the emitter and detector rotate eastwards with the Earth's crust, and the resulting Coriolis effect (in optics also known as Sagnac effect) should be taken into account.

The emitter coordinates  $\mathbf{a}$  at CERN and the detector coordinates  $\mathbf{b}$  at OPERA are provided in Table 5 of OPERA Public Note 132 [2] as

$$\mathbf{a} = \begin{pmatrix} a_X \\ a_Y \\ a_Z \end{pmatrix} = \begin{pmatrix} 4\,394\,369.327 \text{ m} \\ 467\,747.795 \text{ m} \\ 4\,584\,236.112 \text{ m} \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} b_X \\ b_Y \\ b_Z \end{pmatrix} = \begin{pmatrix} 4\,582\,167.465 \text{ m} \\ 1\,106\,521.805 \text{ m} \\ 4\,283\,602.714 \text{ m} \end{pmatrix}.$$

These are Cartesian coordinates in an Earth-centred Earth-fixed (ECEF) coordinate system known as ETRF2000, which is commonly used for geodetic work in Europe.<sup>1</sup>

The distance

$$||\mathbf{b} - \mathbf{a}|| = \sqrt{(b_X - a_X)^2 + (b_Y - a_Y)^2 + (b_Z - a_Z)^2} = 730\,534.610 \text{ m}$$

appears to have formed in [1, p. 10] the basis for the presented neutrino speed calculation, corrected only by local equipment-related delays, distances and calibration parameters at each end.

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<sup>1</sup>The origin of such geodetic XYZ coordinate systems is approximately the center of gravity of Earth, the length unit is 1 m, the Z axis points towards the North Pole, the X axis points towards the prime meridian and surfaces at longitude 0° on the equator off the coast of West Africa, and the Y axis completes a right-handed coordinate system, surfacing at longitude 90° east on the equator in the Indian Ocean.

At the speed of light  $c$ , a particle travels that distance in about

$$\Delta t = ||\mathbf{b} - \mathbf{a}||/c = 2.43680116 \text{ ms.}$$

The Earth rotates eastwards around the Z axis, completing one revolution per stellar day of about 23 h 56 min 4 s = 86 164 s, resulting in an angular frequency of about

$$\omega = 2\pi/86\,164 \text{ s} = 72.921 \text{ } \mu\text{rad/s.}$$

During the neutrino's time-of-flight  $\Delta t$ , Earth will rotate by an angle of

$$\phi = \omega \cdot \Delta t = 177.695 \text{ nrad.}$$

We now define a new coordinate system IRF that is identical to ETRF2000 at the time instant when the neutrino leaves the emitter location, but does not rotate. For the distance calculation in IRF, the emitter coordinates  $\mathbf{a}$  remain the same as in ETRF2000. However, the receiver coordinates  $\mathbf{b}$  at the time of arrival change in IRF to

$$\mathbf{b}' = \begin{pmatrix} \cos \phi & -\sin \phi & 0 \\ \sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \mathbf{b}$$

because ETRF2000 rotated eastwards around its Z axis during the neutrino's flight duration  $\Delta t$  by angle  $\phi$ , and with it the OPERA detector, by  $||\mathbf{b}' - \mathbf{b}|| = 0.838 \text{ m}$ .

In the non-rotating coordinate system IRF, the distance traveled by the neutrino changes to

$$||\mathbf{b}' - \mathbf{a}|| = 730\,533.949 \text{ m} = ||\mathbf{b} - \mathbf{a}|| + 0.661 \text{ m.}$$

Therefore, without taking into account the Coriolis/Sagnac effect that emerges when the neutrino is observed in a rotating coordinate system, the neutrino would appear to arrive 0.661 m behind of where it should be if it traveled at the speed of light, or 2.2 ns late.<sup>2</sup> This actually adds to the 60 ns measurement reported in [1].

## Conclusion

In the absence of the keywords “inertial”, “rotation”, “Coriolis”, “Sagnac” in [1, 2], it seemed prudent to verify what impact a failure to perform the distance calculation in a non-rotating coordinate system would have on the observed time of flight. The result, a 2.2 ns later arrival, should be taken into account<sup>3</sup>, but only strengthens the statistical significance of the reported measurements.

## References

- [1] T. Adam, et al.: Measurement of the neutrino velocity with the OPERA detector in the CNGS beam. 22 September 2011, arXiv:1109.4897v1 [hep-ex]
- [2] G. Colosimo, et al.: Determination of the CNGS global geodesy. 22 September 2011  
<http://operaweb.lngs.infn.it/Opera/publicnotes/note132.pdf>

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<sup>2</sup>Thanks to John Field (CERN) for pointing out a sign error in the first version of this note (3 October 2011).

<sup>3</sup>The authors of [2] have since released Version 2 (10 October 2011), where section 7.3 now addresses this.